Superconductivity from a non-Fermi-liquid metal: Kondo fluctuation mechanism in slave-fermion theory

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We propose Kondo fluctuation mechanism of superconductivity, differentiated from the spin-fluctuation theory as the standard model for unconventional superconductivity in the weak-coupling approach. Based on the U(1) slave-fermion representation of an effective Anderson lattice model, where localized spins are described by the Schwinger boson theory and hybridization or Kondo fluctuations weaken antiferromagnetic correlations of localized spins, we found an antiferromagnetic quantum critical point from an antiferromagnetic metal to a heavy-fermion metal in our recent study. The Kondo-induced antiferromagnetic quantum critical point was shown to be described by both conduction electrons and fermionic holons interacting with critical spin fluctuations given by deconfined bosonic spinons with a spin quantum number 1/2. Surprisingly, such critical modes turned out to be described by the dynamical exponent $z=3$, giving rise to the well-known non-Fermi-liquid physics such as the divergent Grüneisen ratio with an exponent 2/3 and temperature-linear resistivity in three dimensions. We find that the $z=3$ antiferromagnetic quantum critical point becomes unstable against superconductivity, where critical spinon excitations give rise to pairing correlations between conduction electrons and between fermionic holons, respectively, via hybridization fluctuations. Such two kinds of pairing correlations result in multigap unconventional superconductivity around the antiferromagnetic quantum critical point of the slave-fermion theory, where *s-wave* pairing is not favored generically due to strong correlations. We show that the ratio between each superconducting gap for conduction electrons Δ_c and holons Δ_f and the transition temperature T_c is $2\Delta_c/T_c \sim 9$ and $2\Delta_f/T_c \sim \mathcal{O}(10^{-1})$, remarkably consistent with CeCoIn$_5$. A fingerprint of the Kondo mechanism is emergence of two kinds of resonance modes in not only spin but also charge fluctuations, where the charge resonance mode at an antiferromagnetic wave vector originates from *d*-wave pairing of spinless holons. We discuss how the Kondo fluctuation theory differs from the spin-fluctuation approach.

I. INTRODUCTION

Superconductivity from non-Fermi liquids (NFLs) has been one of the central problems in modern condensed-matter physics, associated with high-T_c cuprates[1] and heavy-fermion (HF) quantum critical points (QCPs),[2] where the theory of superconductivity (SC) Bardeen-Cooper-Schrieffer (BCS) mechanism[3] needs to be generalized, depending on the nature of the normal state. When phonons are introduced in normal metals described by Fermi liquid, where Coulomb interactions are screened to become local allowing coherent electrons, there appear attractive interactions between quasiparticles within the time scale given by the Debye frequency associated with relaxation of ions. Then, such attractions give rise to an instability of the whole Fermi surface, causing superconductivity. On the other hand, effective interactions between electrons are not screened completely and still long ranged around QCPs, causing incoherent electron excitations and showing deviation of Fermi-liquid physics. A natural question is what happens if phonon excitations are introduced into such a non-Fermi-liquid metal. Will attractive interactions emerge against such long-range interactions? Although fully self-consistent diagrammatic analysis has not been performed yet as far as we know, it would not be easy to generate such attractions due to phonons.

The first point is on the nature of the non-Fermi-liquid phase, the hallmark of which is beyond the T^2 electrical re-

sistivity, for example, a typical T-linear behavior in various heavy-fermion critical metals. A phenomenological description was proposed, so called the marginal Fermi-liquid ansatz, where electron quasiparticles decay into bunch of particle-hole excitations, regarded as an example of orthogonality catastrophe.[4] One possible origin is quantum criticality, where scattering with critical fluctuations gives rise to the $\omega \ln \omega$ self-energy with frequency ω. Then, the final question is whether such critical fluctuations as the source of non-Fermi-liquid physics will cause superconducting instability in marginal Fermi liquid, where phonon excitations are not expected to play an important role around QCPs.

The so-called spin-fluctuation scenario has been regarded as the standard model for superconductivity out of non-Fermi liquids, where the quantum critical normal state is described by the Hertz-Moriya-Millis theory with the dynamical exponent z,[5] resulting in the temperature-linear resistivity in two dimensions when $z=2$ and effective interactions are oscillatory in space, allowing unconventional pairing beyond s wave.[6,7] This Fermi-liquid-based theory is quite parallel with the BCS theory, where phonons are replaced with antiferromagnetic (AF) spin fluctuations and Migdal theorem[8] holds in both mechanism. A fingerprint of this nonphonon mechanism is emergence of spin-resonance modes at an antiferromagnetic wave vector in the superconducting phase, analogous with linewidth narrowing of the phonon spectrum at frequency below twice of the supercon-

ducting gap, actually measured in both high-T_c cuprates[6] and heavy-fermion superconductors.[7]

Recently, several heavy-fermion compounds have been shown not to follow the spin-fluctuation scenario.[9–13] Anomalous thermodynamics such as the divergent Grüneisen ratio with an exponent 2/3 (Ref. 9) and non-Fermi-liquid transport of temperature-linear resistivity in three dimensions[10] turn out to be beyond the description of the Fermi-liquid-based theory.[14,15] Both divergence of the effective mass near the QCP (Ref. 11) and the presence of localized magnetic moments at the transition toward magnetism[12] seem to support a more exotic scenario. In addition, rather large entropy and small magnetic moments in the antiferromagnetic phase may be associated with antiferromagnetism out of a spin-liquid Mott insulator.[16] Combined with the Fermi-surface reconstruction at the QCP,[11,13] this quantum transition is assumed to show breakdown of the Kondo effect as an orbital selective Mott transition,[17–19] where only the f electrons experience the metal-insulator transition.

The above discussion implies that superconductivity from such an anomalous quantum critical metal is difficult to understand within the spin-fluctuation theory. Actually, superconductivity around the antiferromagnetic QCP of CeRhIn$_5$ was claimed to be beyond the spin-fluctuation framework because this antiferromagnetic QCP seems to be "local" associated with breakdown of the Kondo effect, supported from the sublinear-in-temperature electrical resistivity and isotropic scattering emerging around the QCP, but not in the heavy-fermion phase.[20] Multigap unconventional superconductivity was proposed in CeCoIn$_5$, where large gap coexists with small gap associated with various Fermi surfaces,[21] requiring a new kind of theoretical framework for superconductivity around the QCP.

In the theoretical point of view two kinds of heavy-fermion QCPs were proposed, where nature of spin dynamics is at the heart of heavy-fermion quantum criticality.[22] The Ruderman-Kittel-Kasuya-Yosida (RKKY) induced antiferromagnetic QCP is nothing but the Stoner instability of heavy quasiparticles and only small pieces of Fermi surface become critical via nesting. Superconductivity out of this QCP is described by the spin-fluctuation mechanism. On the other hand, the Kondo-induced QCP leads the whole Fermi surface to be critical, associated with formation of heavy quasiparticles. Recently, a dynamical mean-field theory study has shown that the Kondo-induced QCP may be identified with an orbital selective Mott transition,[19] where spins become localized in the antiferromagnetic phase in contrast with the RKKY mechanism. Importance of Gutzwiller projection was emphasized in the Mott limit of one-band Hubbard model.[23] In this respect the weak-coupling approach is difficult to apply to the strong-coupling problem. An important question is to develop the field theoretic manipulation for Gutzwiller projection.

In this paper we propose Kondo fluctuation mechanism of superconductivity, differentiated from the spin-fluctuation theory. Based on the U(1) slave-fermion representation of an effective Anderson lattice model, where localized spins are described by the Schwinger boson theory[24] and hybridization or Kondo fluctuations weaken antiferromagnetic correlations of localized spins, we found an antiferromagnetic QCP from

an antiferromagnetic metal to a heavy-fermion metal in our recent study.[25] The Kondo-induced antiferromagnetic QCP was shown to be described by both conduction electrons and fermionic holons interacting with critical spin fluctuations given by deconfined bosonic spinons with a spin quantum number 1/2. Surprisingly, such critical modes turned out to be described by the dynamical exponent $z=3$, giving rise to the well-known non-Fermi-liquid physics such as the divergent Grüneisen ratio with an exponent 2/3 (Ref. 14) and temperature-linear resistivity in three dimensions.[15] We find that the $z=3$ antiferromagnetic QCP becomes unstable against superconductivity, where critical spinon excitations give rise to pairing correlations between conduction electrons and between fermionic holons, respectively, via hybridization fluctuations. Such two kinds of pairing correlations result in multigap unconventional superconductivity around the antiferromagnetic QCP of the slave-fermion theory, where s-wave pairing is not favored generically due to strong correlations. We show that the ratio between each superconducting gap for conduction electrons Δ_c and holons Δ_f and the transition temperature T_c is $2\Delta_c/T_c \sim 9$ and $2\Delta_f/T_c \sim \mathcal{O}(10^{-1})$, remarkably consistent with CeCoIn$_5$.[21] A fingerprint of the Kondo mechanism is emergence of two kinds of resonance modes in not only spin but also charge fluctuations, where the charge resonance mode at an antiferromagnetic wave vector originates from d-wave pairing of spinless holons. We argue uniqueness and robustness of the Kondo fluctuation mechanism, comparing with other scenarios based on hybridization fluctuations such as the valance-fluctuation,[26] resonating-valance-bond (RVB),[16] and two-channel SU(2) slave-boson[27] theories.

II. U(1) SLAVE-FERMION THEORY OF ANDERSON LATTICE MODEL

A. U(1) slave-fermion representation of an effective Anderson lattice model

We start from an effective Anderson lattice model

$$H_{ALM} = H_c + H_f + H_{Kondo} + H_{RKKY},$$

$$H_c = -t\sum_{\langle ij \rangle} (c_{in\sigma}^\dagger c_{jn\sigma} + \text{H.c.}) - \mu \sum_i c_{in\sigma}^\dagger c_{in\sigma},$$

$$H_f = -\alpha t \sum_{\langle ij \rangle} (d_{in\sigma}^\dagger d_{jn\sigma} + \text{H.c.}) + \epsilon_f \sum_i d_{in\sigma}^\dagger d_{in\sigma},$$

$$H_{Kondo} = V\sum_i (c_{in\sigma}^\dagger d_{in\sigma} + \text{H.c.}),$$

$$H_{RKKY} = \frac{J}{N}\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \qquad (1)$$

exhibiting competition between hybridization fluctuations H_{Kondo} and antiferromagnetic correlations of localized spins H_{RKKY}, where the large-U limit for localized orbitals is taken into account. In particular, RKKY interactions are modeled as effective exchange interactions between localized spins.

We also assume the presence of weak hopping integrals for localized electrons, denoted by $\alpha \ll 1$. Although the hybridization term gives rise to both RKKY interactions in its fourth order ($\sim V^4/U^3$) and hopping integrals in its second order ($\sim V^2/U$),[28] we regard this effective Anderson model as an emergent model of the intermediate energy scale in the renormalization group sense. Here, $\sigma = \uparrow, \downarrow$ represents SU(2) spin and $n = 1, \dots, N$ expresses the number of flavors, allowing us to analyze this model in a systematic way.

Expressing an electron field in a localized orbital as

$$d_{in\sigma} = f_i^\dagger b_{in\sigma}, \tag{2}$$

where f_i carries only charge, called holon, and $b_{in\sigma}$ does only spin, called spinon, the large-U limit in the localized orbital is expressed as the so-called single occupancy constraint,

$$\sum_{n=1}^{N} \sum_{\sigma=\uparrow,\downarrow} b_{in\sigma}^\dagger b_{in\sigma} + f_i^\dagger f_i = 2SN. \tag{3}$$

In the final stage of calculation we will consider $S = 1/2$ and $N = 1$.

Inserting the U(1) slave-fermion representation into the RKKY term, we take

$$\frac{J}{N} \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j = -\frac{J}{N} \sum_{\langle ij \rangle} (\epsilon_{\alpha\beta} b_{in\alpha}^\dagger b_{jn\beta}^\dagger)(\epsilon_{\gamma\delta} b_{im\gamma} b_{jm\delta})$$
$$\rightarrow \sum_{\langle ij \rangle} \left\{ \frac{N}{J} |\Delta_{ij}|^2 - (\Delta_{ij}^* \epsilon_{\sigma\sigma'} b_{in\sigma} b_{jn\sigma'} + \text{H.c.}) \right\} \tag{4}$$

for antiferromagnetic correlations, where Δ_{ij} capture spin-singlet excitations. In the same way we see

$$-\alpha t \sum_{\langle ij \rangle} (d_{in\sigma}^\dagger d_{jn\sigma} + \text{H.c.})$$
$$= -\alpha t \sum_{\langle ij \rangle} (b_{in\sigma}^\dagger f_i f_j^\dagger b_{jn\sigma} + \text{H.c.})$$
$$\rightarrow \alpha t \sum_{\langle ij \rangle} \{ (\chi_{ij}^{b*} \chi_{ij}^f + \text{H.c.}) - (f_i \chi_{ij}^{b*} f_j^\dagger + \text{H.c.})$$
$$- (b_{in\sigma}^\dagger \chi_{ij}^f b_{jn\sigma} + \text{H.c.}) \}, \tag{5}$$

where χ_{ij}^b keep hopping fluctuations for holons and χ_{ij}^f take ferromagnetic correlations.

Based on Eqs. (4) and (5), we obtain an effective Lagrangian in the U(1) slave-fermion representation of the Anderson lattice model

$$Z = \int Dc_{in\sigma} Db_{in\sigma} Df_i D\Delta_{ij} D\chi_{ij}^b D\chi_{ij}^f D\lambda_i e^{-\int_0^\beta d\tau L},$$

$$L = L_c + L_f + L_b + L_V + L_0,$$

$$L_c = \sum_i c_{in\sigma}^\dagger (\partial_\tau - \mu) c_{in\sigma} - t \sum_{\langle ij \rangle} (c_{in\sigma}^\dagger c_{jn\sigma} + \text{H.c.}),$$

$$L_f = \sum_i f_i^\dagger (\partial_\tau + i\lambda_i) f_i + \alpha t \sum_{\langle ij \rangle} (f_j^\dagger \chi_{ij}^{b*} f_i + \text{H.c.}),$$

$$L_b = \sum_i b_{in\sigma}^\dagger (\partial_\tau + \epsilon_f + i\lambda_i) b_{in\sigma} - \alpha t \sum_{\langle ij \rangle} (b_{in\sigma}^\dagger \chi_{ij}^f b_{jn\sigma} + \text{H.c.})$$
$$- J \sum_{\langle ij \rangle} (\Delta_{ij}^* \epsilon_{\sigma\sigma'} b_{in\sigma} b_{jn\sigma'} + \text{H.c.}),$$

$$L_V = V \sum_i (c_{in\sigma}^\dagger b_{in\sigma} f_i^\dagger + \text{H.c.}),$$

$$L_0 = \alpha t \sum_{\langle ij \rangle} (\chi_{ij}^{b*} \chi_{ij}^f + \text{H.c.}) + NJ \sum_{\langle ij \rangle} |\Delta_{ij}|^2 - i \sum_i 2NS\lambda_i, \tag{6}$$

where the hybridization term V competes with the antiferromagnetic correlation term J for localized electrons, modeled as the nearest-neighbor spin-exchange interaction. L_c describes dynamics of conduction electrons $c_{in\sigma}$, where μ and t are their chemical potential and kinetic energy, respectively. L_f and L_b govern dynamics of localized electrons, decomposed with fermionic holons f_i and bosonic spinons $b_{in\sigma}$, where local antiferromagnetic correlations Δ_{ij} are introduced in the Sp(N) representation for the spin-exchange term J with an index $n = 1, \dots, N$ (Ref. 24) and an almost flat band with $\alpha \ll 1$ is allowed[18] to describe hopping of holons χ_{ij}^b and spinons χ_{ij}^f, respectively. ϵ_f is an energy level for the flat band and λ_i is a Lagrange multiplier field to impose the slave-fermion constraint. L_V is the hybridization term, involving conduction electrons, holons, and spinons. L_0 represents condensation energy with $N = 1$ and $S = 1/2$ in the physical case.

In the decoupling limit of $V \rightarrow 0$ the slave-fermion Lagrangian is reduced to two decoupled sectors for conduction electrons and spinons, where ferromagnetic correlations χ_{ij}^f vanish due to $\langle f_i^\dagger f_i \rangle = 0$ in the spinon sector, recovering the Schwinger-boson theory for the half-filled quantum antiferromagnet[24]

$$Z = \int Dc_{in\sigma} Db_{in\sigma} D\Delta_{ij} D\lambda_i e^{-\int_0^\beta d\tau(L_c + L_b)},$$

$$L_c = \sum_i c_{in\sigma}^\dagger (\partial_\tau - \mu) c_{in\sigma} - t \sum_{\langle ij \rangle} (c_{in\sigma}^\dagger c_{jn\sigma} + \text{H.c.}),$$

$$L_b = \sum_i b_{in\sigma}^\dagger (\partial_\tau + \epsilon_f + i\lambda_i) b_{in\sigma} - \sum_{\langle ij \rangle} (\Delta_{ij}^* \epsilon_{\sigma\sigma'} b_{in\sigma} b_{jn\sigma'} + \text{H.c.})$$
$$+ \frac{N}{J} \sum_{\langle ij \rangle} |\Delta_{ij}|^2 - i \sum_i 2NS\lambda_i. \tag{7}$$

Actually, this is our starting point for the description of localized spins instead of itinerant electrons in the Hertz-Moriya-Millis theory. In this respect the present problem generalizes the Schwinger-boson theory, turning on hybridization fluctuations to cause "hole doping" in the localized band, represented by fermionic holons f_i. Particularly, hybridization fluctuations give rise to ferromagnetic correlations via effective hole doping, weakening antiferromagnetic correlations Δ_{ij} and destroying the antiferromagnetic order $\langle b_{in\sigma} \rangle = 0$.
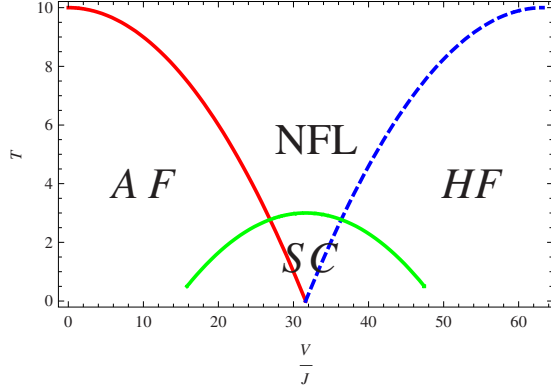
FIG. 1. (Color online) Schematic phase diagram for hybridization-fluctuation-induced d-wave superconductivity around the $z=3$ AF QCP with an AF transition temperature (thick-left), crossover temperature to the heavy-fermion (HF) phase (dashed-right), SC transition temperature (thick-center dome), and NFL, where both the red thick and blue dashed lines were found in Ref. 25.

B. $z=3$ antiferromagnetic quantum critical metal

In the recent study we performed the mean-field analysis with uniform hopping $\chi_{ij}^{f(b)} \to \chi_{f(b)}$, pairing $\Delta_{ij} \to \Delta$, and chemical potential $i\lambda_i \to \lambda$, and found the slave-fermion mean-field phase diagram for the Anderson lattice model (Fig. 1).[25] The antiferromagnetic long-range order turns out to vanish at the critical hybridization strength V_c but short-range antiferromagnetic correlations still exist at the QCP. In the antiferromagnetic phase ($V \ll V_c$) band hybridization is allowed, but the area of the Fermi surface is small, proportional to δ, the density of conduction electrons, because the effective chemical potential of holons is almost on the top of the holon band and the density of holons is vanishingly small. Enhancing the hybridization coupling constant ($V > V_c$), the holon chemical potential shifts to the lower part, filling holons into the flat band and causing heavy fermions. In this description the heavy-fermion transition at finite temperatures turns into crossover, where the crossover temperature T_{FL} is given by gap of spinon excitations $T_{FL} \sim \xi_s^{-1}$ with the correlation length $\xi_s = [(\lambda - 2d\alpha t\chi_f)^2 - (2d\Delta)^2]^{-1/2}$ since scattering of conduction electrons and holons with spinon fluctuations is suppressed below this temperature allowing Fermi-liquid physics.

Fluctuation corrections are taken into account for quantum critical physics in the Eliashberg framework, where vertex corrections are neglected.[17,18] Our main discovery was that dynamics of spinon fluctuations is described by $z=3$ critical theory due to Landau damping of electron-holon polarization above an intrinsic energy scale E^* while by $z=1$ O(4) nonlinear σ model below E^*.[25] The energy scale $E^* \propto \alpha D(q^*/k_F^c)^3$ originates from the mismatch $q^* = |k_F^f - k_F^c|$ of the Fermi surfaces of the conduction electrons k_F^c and holons k_F^f, shown to vary from $\mathcal{O}(10^0)$ to $\mathcal{O}(10^2)$ mK.[17,18] Actually, inserting the Landau damping self-energy $\Pi_b(q, i\Omega) = \gamma_b \frac{|\Omega|}{q}$ with the damping coefficient $\gamma_b = \frac{\pi}{2} \frac{V^2 \rho_c}{v_F^f}$ into the spinon's full propagator, where ρ_c is the density of states for conduction electrons and v_F^f is the holon velocity, we find their $z=3$ dynamics

TABLE I. Scaling of Grüneisen ratio $\Gamma(T)$, uniform spin susceptibility $\chi(T)$, and resistivity $\rho(T)$ with dynamical z and correlation-length ν exponents in $d=3$ for the slave-fermion theory.

	z & ν	$\Gamma(T)$	$\chi(T)$	$\rho(T)$
SF QCP	3 & 1/2	$T^{-2/3}$	$T^{-2/3}$	$T \ln(2T/E^*)$

$$\Im G_b(q, \Omega) \approx -\frac{\gamma}{2\gamma_b} \frac{\gamma \Omega q}{q^6 + \gamma^2 \Omega^2} \tag{8}$$

with $\gamma \equiv (2\gamma_b)(2d\Delta/v_s^2)$, where $v_s = \sqrt{2[\alpha t\chi_f(\lambda - 2d\alpha t\chi_f) + (2d\Delta^2)]}$ is the velocity of spinons. Then, the correlation-length exponent is given by the usual mean-field value $\nu = 1/2$ since the critical theory is above its upper critical dimension in $d=3$.[29]

Both anomalous thermodynamics and non-Fermi-liquid transport result from the $z=3$ quantum criticality. The so-called Grüneisen ratio, the ratio between the thermal expansion parameter and specific-heat coefficient, diverges with an exponent $\frac{1}{\nu z} = \frac{2}{3}$, where ν is the correlation length exponent.[9,14] The electrical resistivity displays the T-linear behavior in three spatial dimensions,[10,15] different from the $z=2$ spin-density-wave theory ($\sim T^{3/2}$). An important result of the $z=3$ antiferromagnetic QCP in the slave-fermion theory is that the uniform dynamic spin susceptibility diverges with an exponent 2/3, similar to an experiment.[12] Of course, the staggered spin susceptibility diverges as it should be due to the antiferromagnetic instability. Divergence of the uniform spin susceptibility is an inevitable response from the $z=3$ antiferromagnetic QCP. As a result, the $z=3$ antiferromagnetic QCP should be distinguished from the $z=2$ spin-density-wave theory, where physical response functions for the $z=3$ antiferromagnetic QCP are summarized in Table I.

C. Discussion on self-consistency in the slave-fermion theory

One cautious physicist may suspect existence of the $z=3$ antiferromagnetic QCP because such a QCP is based on deconfinement of fractionalized excitations called spinons and holons. If such elementary excitations should be confined from some nonperturbative effects, for example, due to magnetic monopole excitations, the present description becomes illusive. In Sec. II C we discuss that the effective U(1) gauge theory in the slave-fermion representation allows deconfinement of such fractionalized excitations at its QCP, thus this kind of theory is self-consistent in itself.

Before discussing deconfinement of slave particles, we would like to explain the motivation of the slave-particle representation. For clarity, suppose the one-band Hubbard model at half filling without perfect nesting. It is believed that the spin-fluctuation approach is applied to the $u/t < 1$ regime while the Gutzwiller projection should be introduced in $u/t > 1$, simulated with the slave-particle representation. In the latter case the interaction coupling constant u was argued to increase more and more, going to an infinite coupling fixed point.[23] Of course, since this is beyond the perturbative regime and the usual one-loop renormalization

group analysis does not work, this statement is just one claim based on the method of canonical transformation. However, it seems to be true that the Fermi-liquid-based approach is difficult to simulate the Gutzwiller projection. Actually, nobody did not reach the Mott transition regime based on the Fermi-liquid-based approach, where the whole Fermi surface becomes critical while the Fermi-liquid-based theory exhibits its instability of only some parts of the Fermi surface. In the present context the Kondo effect is not reached yet based on the spin-fluctuation approach as far as we know. This is the strong motivation for the slave-particle representation.

To check whether the slave-particle theory is self-consistent or not means to understand whether deconfinement is allowed or not beyond the perturbative analysis. Here, "beyond the perturbative analysis" expresses that magnetic monopole excitations are introduced, allowed in the lattice U(1) gauge theory.[30] Their condensation gives rise to confinement of slave particles, then the present approach loses its physical implication.

It has been known that the pure lattice U(1) gauge theory without matter fields allows deconfinement in three spatial dimensions.[30] More precisely, there is the confinement-deconfinement transition varying the coupling constant, here the internal "electric" charge. In the deconfinement phase introduction of matters strengthens deconfinement and slave particles appear as elementary excitations. An important question is what happens in the confinement phase of the pure gauge theory if we introduce matter fields. This has been regarded as an important issue in the gauge theory approach to strongly correlated electrons. Recently, some reliable arguments have been made.

When matter fields are gapped, confinement survives, of course. The question is what happens when matter fluctuations are critical or gapless. Hermele *et al.*[31] claimed that magnetic monopole excitations can be suppressed when there are plenty of flavors for Dirac fermions in QED$_3$ (quantum electrodynamics in two space and one time dimensions). More precisely, they showed that the scaling dimension of the monopole excitation operator is given by the flavor number N of Dirac fermions at the infrared stable fixed point so called the algebraic spin liquid. At the fixed point of QED$_3$ they calculated energy of one magnetic monopole, proportional to the fermion flavor number N. Based on the state-operator correspondence of the conformal field theory, such an energy is identified with the scaling dimension of the magnetic monopole insertion operator. Since it is proportional to N, monopole excitations become irrelevant in the large N limit. This relativistic study was extended to the nonrelativistic case, where there is a Fermi surface of spinons.[32,33] Since there are plenty of fermions around the Fermi surface, one may expect that deconfinement always occurs. Actually, it was argued that deconfinement indeed happens at the spin-liquid fixed point.[32] A similar result was also obtained in the case of bosonic matters.[34]

In the present U(1) slave-fermion gauge theory we have two kinds of critical matters involved with the internal U(1) gauge charge. These are holons with a Fermi surface and gapless spinons at the antiferromagnetic QCP. In this respect deconfinement is allowed, thus the present theoretical framework is self-consistent at least around the QCP.

Finally, we would like to mention one of the main successes in this approach. Applying the slave-particle representation to the multichannel Kondo problem, one finds the scaling solution of a power law within the so-called noncrossing approximation, identified with a non-Fermi-liquid fixed point due to overscreening. Actually, this physics turns out to coincide with the exact method, the conformal field theory.[35]

III. SUPERCONDUCTIVITY FROM A NON-FERMI-LIQUID METAL

A. Superconducting instability of the $z=3$ antiferromagnetic quantum critical point

First, we show that the $z=3$ antiferromagnetic QCP becomes unstable against unconventional superconductivity, evaluating particle-particle scattering vertices for both conduction electrons and holons with subscripts c and f, respectively,

$$\Phi_{cc}(i\Omega) = -V^2 \frac{1}{\beta} \sum_{i\nu} \sum_l \Phi_{ff}(i\Omega + i\nu) F_b(l, i\nu),$$

$$G_f(k_F^c + l, i\Omega + i\nu) G_f(-k_F^c - l, -i\Omega - i\nu),$$

$$\Phi_{ff}(i\Omega) = -2NV^2 \frac{1}{\beta} \sum_{i\nu} \sum_l \Phi_{cc}(i\Omega + i\nu) F_b(l, i\nu),$$

$$G_c(k_F^f + l, i\Omega + i\nu) G_c(-k_F^f - l, -i\Omega - i\nu). \qquad (9)$$

$\Phi_{cc(ff)}(i\Omega)$ is the particle-particle t matrix for conduction electrons (holons) and

$$G_{c(f)}(k, i\omega) = \frac{1}{i\omega - (\epsilon_k^{c(f)} - \mu_{c(f)}) - \Sigma_n^{c(f)}(i\omega)}$$

is the normal Green's function for conduction electrons (holons), where $\epsilon_k^c = -2t(\cos k_x + \cos k_y + \cos k_z)$ and $\epsilon_k^f = -\alpha \chi_b \epsilon_k^c$ are fermion dispersions, and $\mu_c = \mu$ and $\mu_f = -\lambda$ are their chemical potentials. $\Sigma_n^{c(f)}(i\omega)$ is the normal self-energy of conduction electrons (holons), self-consistently found in the Eliashberg approximation[25]

$$F_b(q, i\Omega) = \frac{-\frac{\epsilon_q^b}{t}\Delta}{-(i\Omega)^2 + [\epsilon_q^b + \epsilon_f + \lambda + \Pi_b(q, i\Omega)]^2 - (\epsilon_q^b/t)^2 \Delta^2}$$

is an anomalous propagator for spinons due to their pairing correlations, shown in H_{RKKY} of Eq. (4), where $\epsilon_q^b = \alpha \chi_f \epsilon_q^c$ is the spinon bare dispersion. $\Pi_b(q, i\Omega)$ is the normal self-energy given by the Landau damping form, as discussed before. The presence of the anomalous spinon propagator or antiferromagnetic correlations is an important ingredient for the Kondo fluctuation mechanism, discussed in more detail later (Fig. 2). The negative sign in the right-hand side implies that s-wave superconductivity is prohibited as expected due to strong correlations.

In the long wavelength and low energy limits the anomalous spinon Green's function can be written as
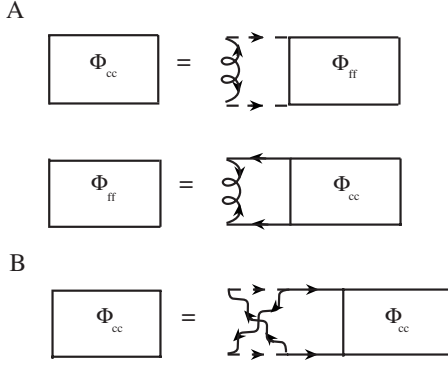
A

B

FIG. 2. (A) Coupled particle-particle t matrices for both conduction electrons and holons in the slave-fermion theory, where the thick line is the electron's Green's function, the dashed line is the holon's Green's function, and the coiling line is the anomalous spinon's Green's function. (B) A particle-particle t matrix for conduction electrons in the slave-boson theory, where the thick line is the electron's Green's function, the dashed line is the spinon's Green's function, and the wavy line is the normal holon's Green's function.

$$\Im F_b(q,\Omega) = \frac{\gamma}{2\gamma_b} \frac{\gamma \Omega q}{q^6 + \gamma^2 \Omega^2}.$$

Inserting this expression into the above and performing momentum integration with the ansatz of d-wave pairing, we obtain

$$\Phi_{cc}(i\Omega) \approx \frac{\mathcal{C}_c^2}{2} \frac{1}{\beta} \sum_{i\nu} \ln\left(\frac{\Omega_c + |\nu - \Omega|}{|\nu - \Omega|}\right) \frac{\Phi_{ff}(i\nu)}{|\nu + i\Sigma_n^f(i\nu)|},$$

$$\Phi_{ff}(i\Omega) \approx 2N \frac{\mathcal{C}_f^2}{2} \frac{1}{\beta} \sum_{i\nu} \ln\left(\frac{\Omega_c + |\nu - \Omega|}{|\nu - \Omega|}\right) \frac{\Phi_{cc}(i\nu)}{|\nu + i\Sigma_n^c(i\nu)|},$$

$$(10)$$

where $z > 1$ ($z = 3$, here) quantum criticality allows the local form for spinon fluctuations with their cutoff frequency Ω_c, and the coupling constants are given by $\mathcal{C}_{c(f)}^2 = \frac{4d\pi V^2 \Delta}{(2\pi)^3 z v_s^2 v_F^{f(c)}}$ with the spinon velocity $v_s = \sqrt{2[\alpha t\chi_f(\lambda - 2d\alpha t\chi_f) + (2d\Delta^2)]}$ (Ref. 25) and holon (conduction electron) Fermi velocity $v_F^{f(c)}$.

Absence of quasiparticles at the $z = 3$ antiferromagnetic QCP is seen from the following fermion self-energies

$$i\Sigma_n^c(i\omega) = g_c^2 \omega \ln\frac{\Omega_c}{|\omega|}, \quad i\Sigma_n^f(i\omega) = g_f^2 \omega \ln\frac{\Omega_c}{|\omega|}, \quad (11)$$

where $g_c^2 = \frac{dV^2\Delta}{6\pi^2 v_s^2 v_F^c}$ and $g_f^2 = 2N\frac{dV^2\Delta}{6\pi^2 v_s^2 v_F^c}$.[14,15] Although this corresponds to the marginal Fermi-liquid ansatz, its mechanism in the strong-coupling approach differs from the spin-fluctuation scenario. Inserting these non-Fermi-liquid self-energies, Eq. (10) can be written as follows:

$$\Phi_{cc}(i\Omega) \approx \mathcal{C}_c^2 \int_{T_c}^{\infty} d\nu \frac{\Phi_{ff}(i\nu)}{\nu\left(1 + g_f^2 \ln\frac{\Omega_c}{\nu}\right)} \ln\frac{\Omega_c}{\sqrt{|\nu^2 - \Omega^2|}},$$

$$\Phi_{ff}(i\Omega) \approx 2N\mathcal{C}_f^2 \int_{T_c}^{\infty} d\nu \frac{\Phi_{cc}(i\nu)}{\nu\left(1 + g_c^2 \ln\frac{\Omega_c}{\nu}\right)} \ln\frac{\Omega_c}{\sqrt{|\nu^2 - \Omega^2|}},$$

$$(12)$$

where finite-temperature effects are introduced as the lower cutoff approximately.[36] Following the procedure of Ref. 36, we find

$$T_c \approx \Omega_c e^{-\pi/\sqrt{\sqrt{2N}\mathcal{C}_c\mathcal{C}_f}} \quad (13)$$

in the "BCS" limit $g_{c(f)}^2 \lesssim 1$. An important lesson in this expression is that the $1/\sqrt{\mathcal{C}_c\mathcal{C}_f} \propto 1/V$ factor in the exponential appears instead of $1/V^2$ associated with the absence of quasiparticles. Using appropriate parameters shown to fit thermodynamics of YbRh$_2$Si$_2$ qualitatively well,[14] we see that T_c varies from $\mathcal{O}(10^0)$ to $\mathcal{O}(10^1)$ K depending on $10 \leq \Omega_c \leq 30$ K, consistent with Ce(Co,Rh)In$_5$.[20]

B. Kondo fluctuation mechanism: Multigap superconductivity

To understand the d-wave superconductivity around the $z = 3$ deconfined antiferromagnetic QCP, we develop an Eliashberg theory[6] for the hybridization-induced superconductivity. The Luttinger-Ward functional can be constructed as $Y_{LW} = Y_{LW}^N + Y_{LW}^S$ with

$$Y_{LW}^N = 2NV^2 \frac{1}{\beta} \sum_{i\Omega} \sum_q \frac{1}{\beta} \sum_{i\omega} \sum_k G_c(k+q, i\omega + i\Omega)$$

$$\times G_b(q, i\Omega) G_f(k, i\omega),$$

$$Y_{LW}^S = -2NV^2 \frac{1}{\beta} \sum_{i\Omega} \sum_q \frac{1}{\beta} \sum_{i\omega} \sum_k F_c(k+q, i\omega + i\Omega)$$

$$\times F_b(q, i\Omega) F_f(k, i\omega), \quad (14)$$

where Y_{LW}^N is for normal self-energies with each normal Green's function and Y_{LW}^S is for anomalous self-energies with each anomalous propagator[29]

$$G_{c(f)}(k, i\omega) = \frac{i\omega - \Sigma_n^{c(f)}(i\omega) + (\epsilon_k^{c(f)} - \mu_{c(f)})}{[i\omega - \Sigma_n^{c(f)}(i\omega)]^2 - (\epsilon_k^{c(f)} - \mu_{c(f)})^2 - \Sigma_p^{c(f)2}(k, i\omega)}$$

is the normal electron (holon) Green's function with the d-wave pairing anomalous self-energy $\Sigma_p^{c(f)}(k, i\omega)$, and $G_b(q, i\Omega)$ is the normal spinon propagator, where its anomalous self-energy can be neglected as long as it is smaller than its pairing order Δ,

$$F_{c(f)}(k, i\omega) = \frac{\Sigma_p^{c(f)}(k, i\omega)}{[i\omega - \Sigma_n^{c(f)}(i\omega)]^2 - (\epsilon_k^{c(f)} - \mu_{c(f)})^2 - \Sigma_p^{c(f)2}(k, i\omega)}$$

is the anomalous electron (holon) Green's function, and the anomalous spinon propagator is the same as before because the presence of antiferromagnetic correlations Δ allows us to neglect its anomalous self-energy.

The electron and holon pairing self-energies are given by

$$\Sigma_p^c(k_F^c, i\omega) = \frac{V^2}{2\pi v_F^f} \frac{1}{\beta} \sum_{i\Omega} \frac{\Sigma_p^f(i\Omega) F_b(i\Omega - i\omega)}{\sqrt{[\Omega + i\Sigma_n^f(i\Omega)]^2 + \Sigma_p^{f2}(i\Omega)}},$$

$$\Sigma_p^f(k_F^f, i\omega) = \frac{2NV^2}{2\pi v_F^c} \frac{1}{\beta} \sum_{i\Omega} \frac{\Sigma_p^c(i\Omega) F_b(i\Omega - i\omega)}{\sqrt{[\Omega + i\Sigma_n^c(i\Omega)]^2 + \Sigma_p^{c2}(i\Omega)}},$$

(15)

where d-wave pairing is assumed in the sign of the anomalous fermion self-energy and $F_b(i\Omega) = \int \frac{d^{d-1}q_\perp}{(2\pi)^{d-1}} F_b(q_\perp, i\Omega)$. This expression is consistent with Eq. (10), justifying our derivation of Eliashberg equations for pairing self-energies.

It is valuable to find the BCS limit of these equations appropriate for the "weak" coupling case. We obtain coupled BCS equations for electron and holon pairing order parameters

$$\Delta_c = \mathcal{B}_c \int_0^{\Omega_c} d\xi \frac{\Delta_f}{\sqrt{\xi^2 + \Delta_f^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_f^2}}{2T},$$

$$\Delta_f = 2N\mathcal{B}_f \int_0^{\Omega_c} d\xi \frac{\Delta_c}{\sqrt{\xi^2 + \Delta_c^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_c^2}}{2T}, \quad (16)$$

where $\mathcal{B}_{c(f)} = \mathcal{C}_{c(f)}^2 \ln(1 + \frac{v_s^2 \Omega_c^{2/3}}{m_s^2})$ with mass of spinons $m_s^2 \propto \sqrt{(\lambda - 2d\alpha t\chi_f)^2 - (2d\Delta)^2}$ (Ref. 25) in the superconducting state. As a result, we find

$$\frac{2\Delta_c}{T_c} = \mathcal{C}_{BCS} \exp\left(-\frac{\mathcal{V}_0^{-1}}{2N\mathcal{B}_f} + \frac{1}{\sqrt{2N\mathcal{B}_f\mathcal{B}_c}}\right),$$

$$\frac{2\Delta_f}{T_c} = \mathcal{C}_{BCS} \exp\left(-\frac{\mathcal{V}_0}{\mathcal{B}_c} + \frac{1}{\sqrt{2N\mathcal{B}_f\mathcal{B}_c}}\right), \quad (17)$$

where $\mathcal{V}_0 = \frac{\Delta_c}{\Delta_f}$ is determined by

$$\frac{\mathcal{V}_0}{\mathcal{B}_c} - \frac{\mathcal{V}_0^{-1}}{2N\mathcal{B}_f} = \ln \mathcal{V}_0 \quad (18)$$

and $\mathcal{C}_{BCS} \approx 3.5$ is the BCS value. Within the range of T_c given by Eq. (13), we obtain $2\Delta_c/T_c \approx 2.7\mathcal{C}_{BCS} \sim 9$ while $2\Delta_f/T_c \sim \mathcal{O}(10^{-1})$.

Recently, thermal conductivity experiments on the heavy-fermion superconductor CeCoIn$_5$ down to 10 mK revealed strong multigap effects with a remarkably low "critical" field for the small gap band, showing that the complexity of heavy-fermion band structure has a direct impact on their response under magnetic field.[21] In particular, the small gap is claimed to originate from light electrons instead of heavy fermions, combined with previous measurements. This aspect seems to be not consistent with the present description, where such a small gap appears from pairing correlations of heavy fermions, holons, although the gap to critical temperature ratio, i.e., $2\Delta_c/T_c \approx 2.7\mathcal{C}_{BCS} \sim 9$ and $2\Delta_f/T_c \sim \mathcal{O}(10^{-1})$ matches with CeCoIn$_5$.[21] In the Kondo fluctuation mechanism it seems to be natural that the small gap arises from heavy fermions. We believe that this point should be clarified

in experiments, particularly, from the measurement for CeRhIn$_5$, where the pairing glue in this superconducting material is claimed to be some local excitations associated with Kondo fluctuations.[20]

C. A fingerprint of the Kondo fluctuation mechanism

The hallmark of the spin-fluctuation-induced d-wave superconductivity was argued to be emergence of the spin-resonance mode at an antiferromagnetic wave vector.[6,7] Since the hybridization-induced superconductivity allows the d-wave pairing symmetry, the similar spin-resonance mode is expected to result from pairing correlations of conduction electrons. An important ingredient beyond the spin-fluctuation scenario is d-wave pairing of spinless fermions. We claim that emergence of a charge-resonance mode at an antiferromagnetic wave vector is one fingerprint of the hybridization-induced superconductivity.

We introduce repulsive interactions between nearest-neighbor holons, given by $H_{int}^f = U_f \sum_{\langle ij \rangle} n_i^f n_j^f$, where on-site repulsive interactions do not appear due to the Pauli exclusion principle. Then, the charge susceptibility is given by the standard random-phase-approximation form

$$\chi_c^f(q, i\Omega) = \frac{U_{ff}(q)}{1 - U_{ff}(q)\Pi_c^f(q, i\Omega)}$$

with $U_{ff} = 2U_f \sum_{j=1}^d \cos q_j$. It was shown that $\Im\Pi_c^f(Q, \Omega < 2\Delta_f) = 0$ and it jumps at $\Omega = 2\Delta_f$ as $\Im\Pi_c^f(Q, 2\Delta_f - \epsilon) \neq \Im\Pi_c^f(Q, 2\Delta_f + \epsilon)$ with $\epsilon \to 0$, resulting from d-wave pairing symmetry,[6] where Q is an associated antiferromagnetic wave vector. The presence of jump gives rise to the logarithmic singularity in the real part of the susceptibility as $\Re\Pi_c^f(Q, \Omega) \propto -\Delta_f \ln \frac{2\Delta_f}{|\Omega - 2\Delta_f|}$ via the Kramers-Kronig relation.[37] As a result, the resonance condition of $1 - U_{ff}(Q)\Re\Pi_c^f(Q, \Omega_{res}) = 0$ can be always satisfied, causing a coherent peak in the susceptibility. This is exactly the origin of the spin-resonance mode in the d-wave superconducting state. An important point is that holons do not carry spin quantum numbers but only charge quantum numbers, thus this peak is identified with a charge-resonance mode at the same momentum with the spin-resonance mode. This is an essential prediction of the present mechanism.

IV. DISCUSSION AND SUMMARY

A. Comparison with other theoretical frameworks

An important ingredient in the hybridization-induced mechanism is the presence of an anomalous propagator of spinon excitations associated with antiferromagnetic correlations, allowing the ladder diagram process as the superconducting mechanism (Fig. 2). One can perform the similar t-matrix calculation at the Kondo breakdown QCP of the slave-boson theory. Actually, this was studied in the context of the valance-fluctuation-induced d-wave superconductivity inside the heavy-fermion phase.[26] Extending this mechanism at the Kondo breakdown QCP, one can construct particle-particle t matrices for both conduction electrons and fermionic spinons. An essential difference from the slave-fermion

TABLE II. Mechanism of superconductivity around heavy-fermion QCPs with superconductivity (SC), Fermi liquid (FL), and non-Fermi liquid (NFL).

	SC from NFL	
SC from FL	Weak coupling	Strong coupling
BCS (phonon) mechanism	Spin-fluctuation mechanism	Kondo-fluctuation mechanism

theory is that the pairing channel arises from crossed diagrams instead of ladder diagrams due to the absence of antiferromagnetic correlations, mathematically corresponding to the pairing term of bosonic holons in the slave-boson theory (Fig. 2). Since these crossed diagrams involve momentum integrals, such instability channels become much weaker[8] than those of the slave-fermion theory.

One can modify the valance-fluctuation mechanism at the Kondo breakdown QCP, taking into account not only particle-hole pairs between conduction electrons and fermionic spinons but also their particle-particle pairs. Recently, this was proposed in the SU(2) slave-boson formulation of the uniform mean-field ansatz with two channels for conduction electrons.[27] Another SU(2) formulation is possible in the d-wave pairing ansatz with one channel, basically an extended version of the RVB superconductivity.[16] However, these ideas overestimate quantum fluctuations in spin dynamics, thus have difficulty in describing antiferromagnetism.

B. Robustness of the $z=3$ antiferromagnetic quantum criticality and marginal Fermi-liquid phenomenology

Antiferromagnetism described by the Schwinger boson theory has its characteristic feature, that is, strong ferromagnetic fluctuations when "holes" are doped. Physically, such uniform spin fluctuations result from the fact that the energy dispersion of bosonic spinons has degeneracy at both the ferromagnetic and antiferromagnetic wave vectors. This seems to be an important nature of quantum antiferromagnets, associated with strong quantum fluctuations. Hybridization fluctuations or "Fermi surface" fluctuations give rise to Landau damping, resulting in the $z=3$ antiferromagnetic QCP. Actually, such strong ferromagnetic fluctuations have been observed in the YbRh$_2$Si$_2$-type sample.[38]

If we consider different kinds of orders, an important point is whether the dispersion of bosonic spinons has its minimum at the $q=(0,0,0)$ momentum point or not. If the energy minimum is away from $q=(0,0,0)$, ferromagnetic fluctuations cannot be critical and the Landau damping term will not affect critical spinon dynamics so much because critical spinon excitations appear in different momentum points which cannot feel such damping. In the present problem the $q=(0,0,0)$ point is almost degenerate with the $q=(\pi,\pi,\pi)$ because spinons are in an almost flat band, i.e., $\alpha \ll 1$ in our mathematical expression, thus allowing strong ferromagnetic fluctuations at the antiferromagnetic QCP. This is the key physics for the $z=3$ antiferromagnetic quantum criticality. Suppose a certain $z=3$ QCP in three spatial dimensions. Why does not the marginal Fermi-liquid physics arise in such all systems?

The Fermi-surface problem in higher dimensions than one dimension is extremely difficult. The usually resorted technique so called large N, where N represents the number of fermion flavors, is not well defined in the presence of a Fermi surface, basically originating from bunch of particle-hole soft modes, where all kinds of planar diagrams, not only self-energy corrections but also vertex corrections, should be resumed,[39,40] but of course, we do not know how. In this kind of problems we have two kinds of self-energy corrections. One is the fermion self-energy while the other is the boson self-energy. Although we cannot give a definite answer, it seems that the boson self-energy is determined by the Landau damping form, given by the self-consistent one-loop calculation, Eliashberg theory. Actually, this was checked explicitly in the two-loop order.[39,41] In our opinion this "protection" mechanism may be due to the presence of a Fermi surface. Particle-hole excitations around the Fermi surface would always give rise to the Landau damping around the zero momentum beyond any order. Ironically, the presence of the Fermi surface causes a serious problem to self-energy corrections of fermions. Such calculations in the fermion self-energy require vertex corrections inevitably.[39] Actually, this has been discussed in the community for a long time but there is still no consensus on the explicit expression of the fermion Green's function.[41] In this respect the actual exponent for the fermion self-energy is not known yet.

The main difference from the above problem is that the present problem consists of two bands, where one is normal but the other is almost flat. Although it is not completely confirmed, some arguments are given, associated with the physical reason why vertex corrections can be neglected in the present two-band model.[17,18,42] It is basically due to the fact that the presence of heavy particles allows us to ignore vertex corrections because the coefficient $\alpha \ll 1$ appears in the vertex expression.

In summary, maybe the presence of two bands, more precisely, an almost flat band allows us to consider only self-energy corrections, giving rise to the marginal Fermi-liquid physics. If we are dealing with the one-band problem, we should take into account vertex corrections and we do not know whether the expression of the fermion Green's function is consistent with the marginal Fermi-liquid form or not. Of course, this discussion is based on the assumption that the $z=3$ QCP is stable. Actually, it turns out that the $z=3$ quantum criticality is difficult to be stable if vertex corrections are introduced in the one-band model.[41]

C. Summary

In this paper we found new mechanism of superconductivity from a non-Fermi-liquid metal beyond the spin-

fluctuation framework, originated from strong correlations (Table II). The hybridization mechanism should be regarded robust and unique, where antiferromagnetic correlations play an important role in the presence of hybridization fluctuations at the QCP,[43] implying that the similar Kondo mechanism is difficult to work around the Kondo breakdown QCP in the slave-boson framework. We predicted emergence of the charge resonance mode at an antiferromagnetic wave vector as the fingerprint for the Kondo fluctuation mechanism, resulting from the multigap nature, thus discriminated from the spin-fluctuation scenario. We obtain actual numerical values for the transition temperature and ratio between the superconducting gaps and transition temperature, and find $2\Delta_c/T_c \sim 9$ and $2\Delta_f/T_c \sim \mathcal{O}(10^{-1})$. Although these ratios are consistent with CeCoIn$_5$, the origin of each gap is not compatible with an experiment,[21] where the small gap is claimed to appear from light electrons while it is originated from heavy fermions, holons in the Kondo fluctuation mechanism. We believe that this point should be clarified in experiments, particularly, from the measurement for CeRhIn$_5$, where the mechanism of superconductivity in this material is claimed to differ from that in CeCoIn$_5$.[20]

ACKNOWLEDGMENTS

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